

K-12 Mathematics Learning Progression

Chương trình học Liên Cấp môn Toán học

Kindergarten

Skill Number	Skill	Domain	Skill Statement
1	Know Number Names to 20	Counting & Cardinality	Identify the number name and numeric form of numbers from 0 to 20.
2	Verbally Count to 100	Counting & Cardinality	Count by 1s to 100, count on by 1s starting from any number, and count by 10s from 10 to 100.
3	Count Objects to 20	Counting & Cardinality	Count up to twenty objects and represent number of objects in numeric form.
4	Compare Quantities up to 10	Counting & Cardinality	Use matching and counting strategies to compare the number of objects in two groups and compare two numbers from 0 to 10.
5	Add and Subtract within 10	Algebra & Expressions	Use models to solve addition and subtraction problems within 10.
6	Break Apart Numbers to 10	Algebra & Expressions	Decompose numbers less than or equal to 10 into pairs in more than one way.
7	Make a 10	Algebra & Expressions	Name the quantity that will make 10 when added to a given number of objects. Write an equation to represent the problem.

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8	Numbers 11-19 as 10 Ones and More Ones	Numbers & Operations	Use 10 ones and additional ones to compose and decompose numbers from 11 to 19 to gain foundations for place value.
9	Describe Measurement and Compare	Measurement, Data, & Statistics	Describe measurable attributes of objects. Compare two objects in terms of a common measurable attribute.
10	Classify Objects	Measurement, Data, & Statistics	Classify objects into given categories and sort the categories by count.
11	Name Shapes and Positions	Geometry	Identify and name shapes in the environment and describe shapes as flat or solid. Describe relative positions of objects and shapes.
12	Combine Shapes	Geometry	Use components to build shapes. Combine two- or three-dimensional shapes to create larger shapes.

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1st Grade

Skill Number	Skill	Domain	Skill Statement
1	Read Numbers to 120	Numbers & Operations	Identify the number name and numeric form for numbers from 0 to 120.
2	Count to 120	Numbers & Operations	Count forward by 1s within 120, starting from any number.
3	Identify Numbers 0-99 Using Place Value	Numbers & Operations	Use place value to identify the numeric form that represents the number of objects for 0 to 99 objects.
4	Compare Two-Digit Numbers	Numbers & Operations	Use place value understanding to compare two-digit numbers, recording the comparison with $<$, $>$, or $=$.
5	Add and Subtract to Solve Word Problems	Algebra & Expressions	Add and subtract to solve word problems within 20.
6	Use Strategies to Add and Subtract	Algebra & Expressions	Use mental strategies and the relationship between operations to add and subtract within 20.
7	Use Properties to Add and Subtract	Algebra & Expressions	Use properties of operations to add and subtract within 20.
8	Understand Equality	Algebra & Expressions	Determine whether numerical expressions are equal and complete a number sentence to show equal quantities.
9	Use Models to Add	Numbers & Operations	Use place value and models to add one-digit numbers to two-digit numbers with and without regrouping.

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10	Add and Subtract 10 and Multiples of 10	Numbers & Operations	Use mental math, place value, and models to subtract multiples of 10 from multiples of 10 within 90 or to add multiples of 10 to any two-digit number.
11	Measure and Compare Lengths	Measurement, Data, & Statistics	Understand the concept of length. Order three objects by length. Compare the lengths of two objects indirectly by using a third object. Express length as a whole number of units using shorter objects laid end to end.
12	Tell Time by Hour and Half Hour	Measurement, Data, & Statistics	Use analog and digital clocks to tell time to the nearest hour or half hour.
13	Use Graphs to Solve Problems	Measurement, Data, & Statistics	Represent data with up to three categories using a picture graph or bar graph. Use addition or subtraction to solve one-step problems about data represented in a graph.
14	Describe and Build Shapes	Geometry	Identify a shape based on its defining attributes. Use two- or three-dimensional shapes to build other shapes.
15	Describe Two and Four Equal Shares	Geometry	Partition shapes into equal parts (halves or fourths). Understand that equal parts are smaller than the whole.

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2nd Grade

Skill Number	Skill	Domain	Skill Statement
1	Name Values of Digits to 999	Numbers & Operations	Understand place value in three-digit numbers.
2	Read Numbers in Word Form	Numbers & Operations	Use place value understanding to read and write whole numbers to 1,000 in word form.
3	Read and Write Numbers in Expanded Form	Numbers & Operations	Use place value understanding to read and write whole numbers to 1,000 in expanded form.
4	Compare Three-Digit Numbers	Numbers & Operations	Use place value understanding to compare two three-digit numbers, and record the comparison using $<$, $>$, or $=$.
5	Count and Skip Count to 1,000	Numbers & Operations	Count by 1s, 5s, 10s, and 100s up to 1,000.
6	Add and Subtract within 20	Algebra & Expressions	Fluently add and subtract within 20.
7	Add and Subtract within 100	Numbers & Operations	Use properties and strategies to add and subtract within 100.
8	Use Equations for Word Problems	Algebra & Expressions	Solve one- and two-step word problems. Represent with equations and unknowns.
9	Add and Subtract to Solve Word Problems	Algebra & Expressions	Represent and solve one- and two-step word problems using addition and subtraction within 100.

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10	Add and Subtract within 1,000	Numbers & Operations	Use properties and strategies to add and subtract within 1,000.
11	Foundations of Multiplication	Algebra & Expressions	Determine whether a group of objects is even or odd, relate even numbers to doubles addition. Relate repeated addition to rectangular arrays.
12	Measure Lengths	Measurement, Data, & Statistics	Estimate, measure, and compare lengths of objects using inches, feet, centimeters, or meters.
13	Solve Problems about Length	Measurement, Data, & Statistics	Solve word problems involving lengths using addition, subtraction, number lines, and equations.
14	Tell Time	Measurement, Data, & Statistics	Use digital and analog clocks to tell and write time to the nearest five minutes using a.m. and p.m.
15	Solve Problems about Money	Measurement, Data, & Statistics	Find the value of a group of coins or dollar bills and solve problems involving coins or dollar bills.
16	Solve Problems Using Graphs	Measurement, Data, & Statistics	Use line plots, picture graphs, and bar graphs to solve simple problems involving data.
17	Describe Shapes	Geometry	Use vocabulary for attributes and categories of two- and three-dimensional shapes.
18	Count Squares in Arrays	Geometry	Partition rectangles into rows and columns of equal-sized squares and count the squares as a precursor to understanding the concept of area.

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19	Name Equal Shares	Geometry	Partition circles, squares, and rectangles into two, three, or four equal shares and describe with fraction words.
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3rd Grade

Skill Number	Skill	Domain	Skill Statement
1	Rounding Numbers	Numbers & Operations	Use place value understanding to round numbers to the nearest ten or hundred.
2	Addition and Subtraction	Numbers & Operations	Use place value understanding, the properties of operations, and the relationship between addition and subtraction to fluently add and subtract within 1,000.
3	Model Multiplication and Division	Algebra & Expressions	Interpret whole-number products and quotients. Represent products and quotients using models and expressions. [Limited to products of two whole numbers within 10 and related quotients.]
4	Number Sentences	Algebra & Expressions	Determine the unknown whole number that makes a one-step multiplication or division equation true in mathematical and real-world problems. [Limited to products of two whole numbers within 10 and related quotients.]

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5	Multiplication and Division Properties	Algebra & Expressions	Use properties of multiplication and division, and the inverse relationship between multiplication and division, as strategies to multiply and divide. [Limited to products of two whole numbers within 10 and related quotients.]
6	Multiplication and Division Facts	Algebra & Expressions	Use properties of multiplication and division, and the inverse relationship between multiplication and division, to fluently multiply two numbers within 10 and find related quotients.
7	Real-World Problems	Algebra & Expressions	Use multiplication and division to solve one-step real-world problems.
8	Number Patterns	Algebra & Expressions	Describe, generate, and extend number patterns in mathematical and real-world problems.
9	Two-Step Real-World Problems	Algebra & Expressions	Use the four operations to represent and solve two-step real-world problems.
10	Estimate Solutions	Algebra & Expressions	Use strategies such as rounding and compatible numbers to estimate solutions to mathematical and real-world problems.
11	Multiply by Multiples of 10	Numbers & Operations	Use understanding of multiplication and place value to multiply a one-digit number by a two-digit multiple of 10 to solve mathematical and real-world problems.

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12	Fractions	Fractions & Ratios	Use number lines and area models to represent fractions.
13	Equivalent Fractions	Fractions & Ratios	Use number lines and area models to identify equivalent fractions, and identify fractions that are equivalent to whole numbers.
14	Comparing Fractions	Fractions & Ratios	Use area models and understanding of fraction concepts to compare two fractions with the same numerator or same denominator, recording the comparison with $<$, $>$, or $=$.
15	Time	Measurement, Data, & Statistics	Use analog and digital clocks to tell time to the nearest minute.
16	Time Elapsed	Measurement, Data, & Statistics	Use addition, subtraction, and number line models to solve problems involving elapsed time.
17	Capacity and Mass	Measurement, Data, & Statistics	Estimate mass or capacity, and use the four operations to solve one-step problems involving mass or capacity.
18	Graphs	Measurement, Data, & Statistics	Represent data on a scaled bar graph, scaled picture graph, or line plot. Use addition and subtraction to answer one- and two-step questions about data represented in graphs.

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Chương trình học Liên Cấp môn Toán học

19	Area	Measurement, Data, & Statistics	Understand concepts of area. Use unit squares, addition, and multiplication to find the area of rectangles and figures composed of rectangles.
20	Perimeter	Measurement, Data, & Statistics	Solve real-world and mathematical problems involving perimeter. Identify rectangles that have the same perimeter but different areas, or the same area but different perimeters.
21	Two-Dimensional Shapes	Geometry	Identify attributes of two-dimensional shapes, and understand that shapes in different categories can share attributes.
22	Shape Partitions	Geometry	Use a partitioned shape to build connections between fraction and area concepts.

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4th Grade

Skill Number	Skill	Domain	Skill Statement
1	Place Value	Numbers & Operations	Use place value understanding to compare the value of digits in different places and to read and write multi-digit whole numbers in word form.
2	Expanded Notation	Numbers & Operations	Use place value understanding to read and write multi-digit whole numbers in expanded form.
3	Compare Numbers	Numbers & Operations	Use place value understanding to compare multi-digit whole numbers.
4	Rounding Numbers	Numbers & Operations	Use place value understanding to round multi-digit whole numbers.
5	Addition and Subtraction	Numbers & Operations	Use the standard algorithm to fluently add and subtract multi-digit whole numbers within 1,000,000.

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6	Factors and Multiples	Algebra & Expressions	Use algebraic reasoning to find factors and multiples of a number, and identify whether a number is prime or composite.
7	Multiplication	Numbers & Operations	Use place value understanding, properties of operations, and models to multiply up to a four-digit whole number by a one-digit whole number, or two two-digit whole numbers.
8	Division	Numbers & Operations	Use place value understanding, properties of operations, and models to divide up to a four-digit whole number by a one-digit whole number.
9	Multiplicative Comparisons	Algebra & Expressions	Understand multiplication as a comparison of two quantities, and solve problems involving multiplicative comparisons.
10	Real-World Problems	Algebra & Expressions	Use mathematical reasoning, equations, and estimation strategies to represent and solve multi-step problems involving whole numbers.
11	Patterns	Algebra & Expressions	Use algebraic reasoning and the four operations to identify, generate, and describe number and shape patterns.
12	Equivalent Fractions	Fractions & Ratios	Identify equivalent fractions, and use visual models and numeric reasoning to explain their equivalency. [Limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.]

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13	Compare Fractions	Fractions & Ratios	Use understanding of benchmark fractions, equivalent fractions, and visual models to compare two fractions with different numerators and denominators, recording the comparison with $<$, $>$, or $=$. [Limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.]
14	Adding and Subtracting Fractions	Fractions & Ratios	Add and subtract fractions, including mixed numbers, with common denominators. [Limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.]
15	Multiplying Fractions	Fractions & Ratios	Use models and understanding of multiplication to multiply a fraction by a whole number. Identify equivalent fraction multiplication expressions. Solve problems involving multiplication of a fraction by a whole number. [Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, or 100.]
16	Convert Fractions to Decimals	Fractions & Ratios	Represent fractions with denominators 10 or 100 in decimal form.
17	Compare Decimals	Fractions & Ratios	Compare two decimals to the hundredths place, recording the comparison with $<$, $>$, or $=$.
18	Units of Measurement	Measurement, Data, & Statistics	Know relative sizes of measurement units within the same system, and convert a measurement from a larger unit to a smaller unit.
19	Measuring Objects	Measurement, Data, & Statistics	Use measurement tools with a number line scale to measure objects.

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20	Time	Measurement, Data, & Statistics	Solve problems involving time.
21	Money	Measurement, Data, & Statistics	Solve problems involving money.
22	Perimeter and Area	Measurement, Data, & Statistics	Solve problems involving perimeter and area of rectangles.
23	Represent and Interpret Data	Measurement, Data, & Statistics	Use line plots to represent and interpret data.
24	Points, Lines, Rays, and Segments	Geometry	Recognize and describe points, lines (including parallel and perpendicular), rays, and line segments, and identify them in two-dimensional figures.
25	Angles	Geometry	Recognize and describe acute, obtuse, and right angles, and identify them in two-dimensional figures.
26	Angle Measurement	Measurement, Data, & Statistics	Understand concepts of angles, use a protractor to measure angles, and use addition and subtraction to solve problems involving angle measurements in real-world and mathematical contexts.
27	Classify Two-Dimensional Figures	Geometry	Classify two-dimensional figures based on angle measures, or on the presence or absence of parallel or perpendicular sides.

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28	Symmetry	Geometry	Identify lines of symmetry in figures, and create line-symmetric figures.
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5th Grade

Skill Number	Skill	Domain	Skill Statement
1	Multiply Whole Numbers	Numbers & Operations	Use the standard algorithm to multiply multi-digit whole numbers.
2	Model Division of Whole Numbers	Numbers & Operations	Use place value understanding, properties of operations, the relationship between multiplication and division, and models to divide up to four-digit dividends by two-digit divisors.
3	Numerical Expressions	Algebra & Expressions	Use algebraic reasoning and the order of operations to interpret and evaluate numerical expressions.

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4	Representations of Decimals	Numbers & Operations	Use place value understanding to read and write decimals to the thousandths place in standard form, word form, and expanded form.
5	Comparing Decimals	Numbers & Operations	Use place value understanding to compare decimals to the thousandths place, recording the comparison with $<$, $>$, or $=$.
6	Rounding Decimals	Numbers & Operations	Use place value understanding to round decimals to any place.
7	Powers of Ten	Numbers & Operations	Understand the relationship between adjacent places in the place value system. Use patterns to multiply or divide a number by a power of 10. Use exponents to denote powers of 10.
8	Add and Subtract Decimals	Numbers & Operations	Use place value understanding, properties of operations, the relationship between addition and subtraction, and models to add and subtract decimals to the hundredths place.
9	Multiply and Divide Decimals	Numbers & Operations	Use place value understanding, properties of operations, the relationship between multiplication and division, and models to multiply and divide decimals to the hundredths place.

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10	Real-World Problems	Numbers & Operations	Use the four operations to solve real-world problems involving whole numbers and decimals to the hundredths.
11	Add and Subtract Fractions	Fractions & Ratios	Add and subtract fractions with unlike denominators.
12	Multiplication with Fractions	Fractions & Ratios	Use models, properties of operations, and understanding of multiplication to multiply a fraction or whole number by a fraction. Understand multiplication as scaling, and compare a product to the size of its factors.
13	Division with Fractions	Fractions & Ratios	Interpret a fraction as division. Use models and understanding of division to divide a whole number by a unit fraction or a unit fraction by a whole number.
14	Problem Solving with Fractions	Fractions & Ratios	Use the four operations to solve real-world problems involving fractions. [Division problems limited to whole number divided by unit fraction or unit fraction divided by whole number.]
15	Units of Measure	Measurement, Data, & Statistics	Convert among measurement units within a given system, and use conversions to solve real-world problems.
16	Representing and Interpreting Data	Measurement, Data, & Statistics	Represent fractional data on a line plot. Solve problems involving data presented on a line plot.

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17	Two-Dimensional Figures	Geometry	Understand and use the hierarchical relationship between classes of two-dimensional figures to classify them based on their properties.
18	Volume	Measurement, Data, & Statistics	Understand concepts of volume. Use unit cubes, addition, multiplication, and formulas to find volumes of rectangular prisms and figures composed of rectangular prisms in real-world and mathematical contexts.
19	Coordinate System	Geometry	Understand concepts related to the coordinate plane, and locate and interpret points in the first quadrant of the coordinate plane to solve real-world and mathematical problems.
20	Number Patterns	Algebra & Expressions	Generate two numerical patterns using two given rules, identify the relationship between corresponding terms, represent corresponding terms as ordered pairs, and locate the ordered pairs on the coordinate plane.

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6th Grade

Skill Number	Skill	Domain	Skill Statement
1	Number Theory	Numbers & Operations	Use number reasoning to find the greatest common factor or least common multiple of two numbers, and use these along with the distributive property to generate equivalent expressions.
2	Arithmetic with Decimals	Numbers & Operations	Use standard algorithms to add, subtract, multiply and divide multi-digit decimal numbers.

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3	Division of Whole Numbers	Numbers & Operations	Use the standard algorithm to divide whole numbers, and solve problems involving division.
4	Division of Fractions	Numbers & Operations	Use strategies to divide fractions in real-world and mathematical problems.
5	Negative and Positive Numbers	Numbers & Operations	Describe, compare, and interpret negative and positive rational numbers.
6	Number Lines	Numbers & Operations	Use number lines to describe and compare rational numbers.
7	Absolute Value	Numbers & Operations	Interpret, describe, and compare absolute values of rational numbers.
8	Coordinate Planes	Numbers & Operations	Graph points on a coordinate plane, calculate the distance between two points on a horizontal or vertical line, and understand the locations of points that have been reflected across one or both axes.
9	Ratios and Ratio Language	Fractions & Ratios	Use ratios and ratio language to describe real-world quantities.
10	Unit Rates	Fractions & Ratios	Use ratio reasoning to solve problems involving unit rates.
11	Units of Measurement	Fractions & Ratios	Use ratio reasoning to convert measurement units.

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12	Write Expressions	Algebra & Expressions	Use understanding of operations to read and write numeric and algebraic expressions.
13	Evaluate Expressions	Algebra & Expressions	Use order of operations to evaluate numeric and algebraic expressions, including expressions involving exponents.
14	Equivalent Expressions	Algebra & Expressions	Use properties of operations to identify and generate equivalent algebraic expressions.
15	Solve Equations and Inequalities	Algebra & Expressions	Use substitution to solve one-variable equations and inequalities.
16	Symbolize Problem Situations	Algebra & Expressions	Write expressions with variables to symbolize problem situations.
17	Number Sentences	Algebra & Expressions	Write and solve one-variable equations and inequalities that represent mathematical and real-world situations.
18	Percents	Fractions & Ratios	Solve percent problems.
19	Quantitative Relationships	Algebra & Expressions	Use equations, tables, and graphs to represent quantitative relationships.
20	Area	Geometry	Use area formulas and decomposition to find the areas of triangles, quadrilaterals, and composite figures.

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21	Surface Area and Volume	Geometry	Use nets to represent prisms and pyramids, and find the surface area and volume of prisms and pyramids.
22	Coordinate Geometry	Geometry	Use coordinate graphs to find side lengths and areas of polygons.
23	Statistical Analysis	Measurement, Data, & Statistics	Recognize statistical questions, and use measures of center and variability to describe the distribution of data gathered from a statistical question.
24	Graphing and Interpreting Data	Measurement, Data, & Statistics	Use graphs to display and interpret numerical data.

7th Grade

Skill Number	Skill	Domain	Skill Statement
1	Properties of Addition and Subtraction	Numbers & Operations	Use properties of operations to add and subtract positive and negative rational numbers, and represent these operations on number lines. Understand real-world and mathematical additive inverses.

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2	Properties of Multiplication and Division	Numbers & Operations	Use properties of operations to multiply and divide positive and negative rational numbers in real-world and mathematical problems, and represent these operations on number lines.
3	Compute with Rational Numbers	Numbers & Operations	Add, subtract, multiply, and divide positive and negative rational numbers.
4	Single-Step Real-World Problems	Numbers & Operations	Add, subtract, multiply, and divide positive and negative rational numbers in real-world problems with and without models.
5	Ratios and Proportions	Fractions & Ratios	Use ratios and proportions to solve real-world and mathematical problems.
6	Percents	Fractions & Ratios	Use proportional reasoning to solve real-world multi-step percent problems.
7	Unit Rates	Fractions & Ratios	Solve problems with unit rates computed from verbal descriptions, graphs, tables, and equations, and understand graphs of proportional relationships.
8	Proportional Relationships	Fractions & Ratios	Use proportional reasoning to decide whether two quantities are in a proportional relationship, and create equations and analyze graphs of proportional relationships.

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9	Linear Expressions	Algebra & Expressions	Use properties of operations to find equivalent forms of linear algebraic expressions with rational coefficients, and use equivalent forms to help interpret parts of an expression.
10	Multi-Step Real-World Problems	Algebra & Expressions	Use properties of operations to solve multi-step real-world problems with rational numbers.
11	Symbolize and Solve Equations	Algebra & Expressions	Write and solve one-variable linear equations that represent real-world situations.
12	Symbolize and Solve Inequalities	Algebra & Expressions	Write and solve one-variable linear inequalities that represent real-world situations, and graph solutions on a number line.
13	Triangles	Geometry	Describe and classify triangles with given conditions.
14	Angles	Geometry	Use knowledge of supplementary, complementary, vertical and adjacent angles to find unknown angle measures in a figure.
15	Circles	Geometry	Use formulas for area and circumference of a circle to solve real-world and mathematical problems.

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16	Area, Surface Area, and Volume	Geometry	Use formulas for area, surface area and volume to solve real-world and mathematical problems in two- and three-dimensional composite figures.
17	Three-Dimensional Figures	Geometry	Describe and classify two- and three-dimensional figures with given conditions, and describe cross sections of three-dimensional figures.
18	Scale Drawings	Geometry	Use proportional reasoning to solve problems involving scale drawings of geometric figures.
19	Sampling Analysis	Measurement, Data, & Statistics	Understand sampling methods in surveys and experiments, and make valid generalizations about a population using measures of center or variability of a sample.
20	Central Tendency and Variability	Measurement, Data, & Statistics	Use measures of central tendency and variability to compare datasets, and use these measures to make inferences about a population.
21	Probability	Measurement, Data, & Statistics	Use probability to describe the likelihood of an event. Calculate the theoretical or experimental probability of a simple or compound event, and use it to make predictions. Use tables, lists and tree diagrams to represent sample spaces.

8th Grade

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Skill Number	Skill	Domain	Skill Statement
1	Real Numbers	Numbers & Operations	Identify rational and irrational numbers, find the decimal expansion of a fraction, and convert a decimal expansion to a fraction.
2	Exponential Expressions	Algebra & Expressions	Use the properties of integer exponents to generate equivalent expressions.
3	Square and Cube Roots	Algebra & Expressions	Find the solutions to simple one-variable quadratic and cubic equations, and evaluate perfect squares and perfect cubes.
4	Rational Approximations	Numbers & Operations	Use rational approximation to compare irrational numbers, and estimate the location of an irrational number on a number line.
5	Scientific Notation	Algebra & Expressions	Convert between and use scientific notation and standard form to estimate and compare quantities, and perform operations on numbers written in scientific notation and standard form.
6	Proportional Relationships	Algebra & Expressions	Use tables, graphs and equations to represent, interpret and compare proportional relationships. Use similar triangles to find slope of a non-vertical line in the coordinate plane, and represent lines in the coordinate plane with equations in slope-intercept form.

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7	Solving Linear Equations	Algebra & Expressions	Determine the number of solutions in a one-variable linear equation, and solve one-variable linear equations with rational number coefficients.
8	Systems of Equations	Algebra & Expressions	Find the solution to a system of linear equations algebraically and graphically in real-world and mathematical problems, and describe the number of solutions to a system of linear equations.
9	Functions	Functions	Determine if a set of ordered pairs, a table, a graph, or an equation is a relation and/or a function. Compare properties of functions represented in different ways.
10	Linear vs. Nonlinear	Functions	Determine if a function represented by an equation, graph, table or verbal situation is linear or non-linear, and describe functions qualitatively.
11	Linear Relationships	Functions	Write linear equations in two variables to represent real-world situations, and interpret properties of the linear relationship that it models. Describe behaviors of graphed linear or non-linear functions.
12	Object Transformations	Geometry	Perform transformations of two-dimensional figures in the coordinate plane with translations, rotations, reflections, or dilations.

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13	Similarity and Congruence	Geometry	Recognize when translations, rotations, reflections, or dilations of a two-dimensional figure produce a similar or congruent figure. Use angle-angle criterion to determine when two triangles are similar.
14	Angles and Lines	Geometry	Solve problems involving the interior and exterior angle measures in triangles and the angles formed when parallel lines are cut by a transversal.
15	Pythagorean Theorem	Geometry	Use the Pythagorean Theorem to find missing side lengths in right triangles in real-world and mathematical problems in two- and three-dimensions and to find the distance between two points in the coordinate plane.
16	Volume	Geometry	Use formulas to find the volume of cones, cylinders and spheres in real-world and mathematical problems.
17	Scatter Plots	Measurement, Data, & Statistics	Construct scatter plots to represent bivariate data, and recognize and interpret patterns of association between two quantities.
18	Best-Fit Linear Models	Measurement, Data, & Statistics	Use and interpret a line of best fit to describe the relationship between two quantities on a scatter plot.

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19	Two-Way Tables	Measurement, Data, & Statistics	Recognize, interpret, and complete patterns of association between quantities displayed in a two-way table.
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9th Grade

Skill Number	Skill	Domain	Skill Statement
1	Quantities and Units	Numbers & Operations	Determine the appropriate units and accuracy for quantities in real-world models.
2	Interpret Graphs	Numbers & Operations	Determine and interpret the scale, origin, and accuracy level for graphs and data displays modeling real-world situations.
3	Operations with Rational and Irrational Numbers	Numbers & Operations	Understand and classify the result of adding and multiplying rational and irrational numbers.
4	Rational Exponents	Numbers & Operations	Understand the relationship between rational exponents and radicals. Find equivalent numerical or algebraic expressions involving rational exponents or radicals.
5	Properties of Arithmetic and Equality	Numbers & Operations	Use arithmetic properties of equality to solve linear equations in one variable and justify the solution process.

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6	Solve Linear Equations	Algebra & Expressions	Solve linear equations in one variable including equations with variable coefficients.
7	Rewrite to Solve Linear Variable Equations	Algebra & Expressions	Solve formulas and equations in multiple variables for a defined linear quantity.
8	Solve Linear Inequalities	Algebra & Expressions	Solve linear inequalities in one variable, and graph the solution set on a number line.
9	Symbolize with Linear and Exponential Expressions	Algebra & Expressions	Interpret parts of linear and exponential expressions. Transform exponential expressions using the properties of exponents.
10	Symbolize with Quadratic Expressions	Algebra & Expressions	Interpret parts of quadratic expressions.
11	Create and Solve One-Variable Linear Equations and Inequalities	Algebra & Expressions	Create and solve equations to model linear situations in one variable, and interpret the solution in context.
12	Solve Exponential Equations	Algebra & Expressions	Solve one-variable exponential equations by rewriting with common bases.
13	Create and Solve One-Variable Exponential Equations and Inequalities	Algebra & Expressions	Create and solve equations to model exponential situations in one variable, and interpret the solution in context.

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14	Two-Variable Linear and Exponential Equations	Algebra & Expressions	Write and graph two-variable linear and exponential equations modeling real-world situations.
15	Create Systems of Equations and Inequalities	Algebra & Expressions	Create systems of linear equations and inequalities to model real-world situations, and determine the validity of solutions in context.
16	Systems of Equations	Algebra & Expressions	Solve systems of linear equations graphically or algebraically with substitution and elimination.
17	Linear Inequalities in Two Variables	Algebra & Expressions	Graph and find the solution region of linear inequalities and systems of linear inequalities in mathematical and real-world contexts. Apply optimization techniques to real-world situations.
18	Functions	Functions	Identify functions from graphs, tables, mappings, and ordered pairs. Interpret the relationship between input and output values of functions notation statements.
19	Evaluate Functions	Functions	Evaluate linear and exponential functions given an input, and relate the input and output of functions in real-world situations.
20	Graphing Linear Functions	Functions	Graph linear functions from slope-intercept or point-slope form, and identify slope and intercepts.

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21	Graphing Exponential Functions	Functions	Graph exponential functions, and identify end behavior and intercepts.
22	Linear and Exponential Function Transformations	Functions	Recognize the effects of transformations on linear and exponential functions using graphs, tables, coordinate pairs, and function rules.
23	Solve Equations by Graphing	Functions	Approximate the solution to an equation $f(x) = g(x)$ using graphing, successive approximation, and tables. Identify the solution set of an equation as all points along the graph.
24	Compare Properties of Linear and Exponential Functions	Functions	Compare key features of linear and exponential functions using graphs, tables, verbal descriptions and equations.
25	Interpret Functions in Context	Functions	Graph and interpret key features of linear and exponential functions modeling real-world situations.
26	Domain of a Function	Functions	Identify the domain of linear and exponential functions.
27	Rate of Change	Functions	Calculate and interpret the average rate of change of linear and exponential functions from graphs, tables and equations.
28	Linear and Exponential Models	Functions	Create, interpret, and compare linear and exponential functions given graphs, tables and verbal descriptions.

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29	Writing Functions	Functions	Create functions to model real-world situations by combining linear, exponential, and quadratic functions using arithmetic operations.
30	Sequences	Functions	Model arithmetic and geometric sequences as explicit and recursive functions. Identify terms when given the function defining a sequence.
31	Operations with Polynomial Expressions	Algebra & Expressions	Add, subtract, and multiply linear and quadratic expressions. Describe the set of polynomials as closed under addition, subtraction, and multiplication.
32	Equivalent Quadratic Expressions	Algebra & Expressions	Rewrite polynomial expressions in different forms using factoring methods.
33	Reveal Properties of Quadratic Expressions	Algebra & Expressions	Rewrite quadratic expressions using factoring and complete the square to identify zeros, maxima, and minima.
34	Solve Quadratic Equations	Algebra & Expressions	Find the solutions to quadratic equations using square roots, complete the square, factoring, and the quadratic formula.
35	Create and Solve Quadratic Equations and Inequalities	Algebra & Expressions	Create and solve one and two-variable quadratic equations and inequalities to model real-world relationships.

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36	Rewrite Equations	Algebra & Expressions	Solve formulas and equations in multiple variables for a defined square or radical quantity.
37	Writing Quadratic Functions	Functions	Create quadratic functions representing real-world situations. Combine quadratic functions using addition, subtraction, and multiplication.
38	Interpret Quadratic Functions	Functions	Find and interpret key features of quadratic functions in different forms, including rate of change.
39	Graphing Quadratic Functions	Functions	Manipulate quadratic equations to find intercepts, extreme values, and axes of symmetry. Graph quadratic functions, and interpret key features.
40	Graphing Absolute Value, Step and Piecewise Defined Functions	Functions	Construct graphs and interpret key features of absolute value, step, and piecewise functions.
41	Systems of Linear and Quadratic Equations	Algebra & Expressions	Solve systems of linear and quadratic equations graphically and algebraically.
42	Compare Properties of Functions	Functions	Compare key features of linear, exponential and quadratic functions using graphs, tables, verbal descriptions and equations.

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43	Function Transformations	Functions	Recognize the effects of transformations on quadratic and absolute value functions using graphs, tables, coordinate pairs, and function rules. Recognize patterns of even and odd functions.
44	Inverse Functions	Functions	Find the inverses of linear and simple quadratic functions algebraically for a defined domain.
45	Comparing Data	Measurement, Data, & Statistics	Calculate, interpret, and compare measures of central tendency and spread for data sets. Represent and interpret data on box plots, histograms, and dot plots.
46	Two-Way Frequency Tables	Measurement, Data, & Statistics	Interpret associations between data presented in two-way tables, and make inferences.
47	Scatterplots	Measurement, Data, & Statistics	Interpret, and assess linear, quadratic and exponential functions modeling patterns of association between two quantities. Write function models for data showing linear correlation.

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10th Grade

Skill Number	Skill	Domain	Skill Statement
1	Geometric Definitions	Geometry	Identify and describe definitions of segments, lines, circles, and types of angles.
2	Geometric Constructions	Geometry	Describe the steps for constructing angles, segments, lines, and inscribed figures using a compass and paper folding.
3	Lines - Parallel, Perpendicular, and Ratios	Geometry	Explain and use the relationship between the slopes of parallel and perpendicular lines. Determine the point that divides a segment into a given ratio.
4	Lines and Angles Theorems	Geometry	Understand, use, and justify theorems and proofs about lines, segments and angles, including angles along transversals and angles found in triangles.

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5	Triangle Theorems	Geometry	Understand, use, and justify theorems and proofs of midsegments, perpendicular bisectors, and angles in triangles.
6	Parallelogram Theorems	Geometry	Understand, use, and justify theorems and proofs of parallelograms.
7	Coordinate Geometry	Geometry	Classify quadrilaterals and triangles defined by vertices at coordinate points, and find the areas and perimeters of the figures.
8	Transformations in the Plane	Geometry	Identify, explain and perform transformations of figures on the coordinate plane, including translations, dilations, rotations, and reflections.
9	Congruence	Geometry	Use rigid transformations and congruence criteria to show two figures and corresponding parts are congruent.
10	Similarity and Similarity Transformations	Geometry	Use transformations and similarity criteria to show two figures are similar. Find lengths of sides and angles in similar figures.
11	Similarity and Congruence of Triangles	Geometry	Prove and use relationships between sides and angles of similar or congruent triangles. Find area and perimeter of congruent or similar triangles.

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Chương trình học Liên Cấp môn Toán học

12	Right Triangle Trigonometry	Geometry	Explain and use trigonometric ratios, the Pythagorean theorem, and the relationship between the sides of similar right triangles to solve problems involving right triangles, including real-world situations.
13	Law of Sines and Law of Cosines	Geometry	Understand and use the law of sines and the law of cosines to find the measures of angles and sides of triangles, including triangles used in real-world situations.
14	Area of a Triangle Using Trigonometry	Geometry	Use the trigonometric formula for the area of a triangle, including finding sides or angles given the area.
15	Two and Three-Dimensional Objects	Geometry	Explain how solid figures model real-world figures in two and three dimensions. Identify the cross section of cones, cylinders and prisms.
16	Volume	Geometry	Understand how area and volume formulas are generated, and use the formulas in real-world situations, including calculations with density.
17	Circles without Coordinates	Geometry	Explain how all circles are similar, and use similarity to determine relationships between arc lengths and areas of circles. Construct, explain and use relationships between arcs, chords, tangents, radii, and inscribed angles.

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18	Circles with Coordinates	Geometry	Use the equation for a circle to graph circles. Find the center, radius, points on the curve, area, and circumference of circles defined by the equation.
19	Parabola Equations	Geometry	Use the focus and directrix of a parabola to determine the equation modeling the curve.
20	Independence and Conditional Probability	Measurement, Data, & Statistics	Recognize situations as independent events, and calculate and compare related conditional probabilities, including those displayed in two-way tables.
21	Rules of Probability in Compound Events	Measurement, Data, & Statistics	Describe subsets, unions, intersections, and complements of compound events. Use the addition rule, multiplication rule, permutations and combinations to find probabilities of compound events. Interpret probabilities of compound events to make conclusions.
22	Probability and Decision Making	Measurement, Data, & Statistics	Recognize the fairness of models. Calculate probabilities from data to draw conclusions about real-world situations.

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11th Grade

Skill Number	Skill	Domain	Skill Statement
1	Interpret Polynomial and Rational Expressions	Functions	Interpret parts of polynomial and rational expressions.
2	Operations on Polynomial Expressions	Algebra & Expressions	Add, subtract, and multiply polynomial expressions. Understand that division between polynomials does not always result in a polynomial.
3	Factoring Polynomial Expressions	Algebra & Expressions	Factor higher order polynomials and polynomials in multiple variables. Use algebraic methods to prove polynomial identities.
4	Binomial Theorem	Algebra & Expressions	Use the Binomial Theorem and Pascal's triangle to expand powers of binomial expressions.
5	Simplify Rational Expressions	Algebra & Expressions	Use factoring and properties of exponents to generate equivalent rational expressions.

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6	Polynomial Division	Algebra & Expressions	Use long division and synthetic division to find the quotient of two polynomials.
7	Rational Expressions	Algebra & Expressions	Add, subtract, multiply, and divide rational expressions, and recognize that the result belongs to the set of rational expressions.
8	Geometric Series	Functions	Find the sum of finite geometric series in mathematical and real-world contexts. Use the formula for the sum of a finite geometric series.
9	Complex Numbers	Numbers & Operations	Identify complex numbers. Determine products, sums, differences, and powers of complex numbers.
10	Factor and Solve Polynomial Equations with Complex Solutions	Algebra & Expressions	Solve polynomial equations with complex solutions. Rewrite polynomial expressions as the product of factors containing complex numbers. Understand and use the fundamental principle of algebra.
11	Solve Rational Equations	Algebra & Expressions	Solve rational equations in one variable using algebraic methods, and identify extraneous solutions.
12	Solve Radical Equations	Algebra & Expressions	Solve radical equations in one variable using algebraic methods, and identify extraneous solutions.

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Chương trình học Liên Cấp môn Toán học

13	Rewrite Equations	Algebra & Expressions	Solve polynomial, rational, and radical equations and formulas for a quantity of interest on an appropriate domain.
14	Solve Exponential Models with Logarithms	Functions	Use properties of exponents and logarithms to solve and simplify exponential and logarithmic expressions and equations, including those modeling real-world situations.
15	Create and Solve One-Variable Equations and Inequalities	Algebra & Expressions	Create and solve equations in one variable modeling real-world situations representing polynomial, radical, and rational relationships.
16	Selecting Function Type from Context	Functions	Recognize appropriate function models for real-world situations representing exponential, polynomial, rational, radical, absolute value or trigonometric relationships.
17	Create and Graph Two-Variable Equations	Algebra & Expressions	Write and graph polynomial, radical, and rational equations in two-variables modeling real-world situations.
18	Combining Functions	Functions	Combine exponential, polynomial, rational, and radical functions using addition, subtraction, multiplication and division, including those modeling real-world situations.

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19	Solve Equations by Graphing	Functions	Use technology to graph the solution to an equation $f(x) = g(x)$ where $f(x)$ and $g(x)$ are exponential, radical, rational, polynomial, absolute value, or logarithmic functions.
20	Solve Equations Using Tables	Functions	Use tables to approximate the solution to an equation $f(x) = g(x)$ where $f(x)$ and $g(x)$ are exponential, radical, rational, polynomial, absolute value, or logarithmic functions.
21	Solve Equations Using Successive Approximations	Functions	Use successive approximations to approximate the solution to an equation $f(x) = g(x)$ where $f(x)$ and $g(x)$ are exponential, radical, rational, polynomial, absolute value or logarithmic functions.
22	Create Systems of Equations and Inequalities	Functions	Create systems of equations and inequalities modeling exponential, polynomial, rational, and radical relationships, and evaluate the viability of values in the solution set.
23	Pythagorean Identity	Functions	Prove the Pythagorean identity. Use the Pythagorean identity to find sine, cosine and tangent values for points on the coordinate plane.
24	Unit Circle	Functions	Use the unit circle to find trigonometric values and arc lengths on a circle in mathematical and real-world contexts.

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25	Graphs of Trigonometric Functions	Functions	Graph trigonometric functions. Identify and interpret key features of trigonometric functions in mathematical and real-world contexts.
26	Zeros of Polynomials	Functions	Use the remainder theorem to determine the factors of a polynomial. Identify zeros of a polynomial, and use these to graph the polynomial.
27	Graphing Functions	Functions	Graph nonlinear functions, including nonlinear piecewise functions, from equations and key features.
28	Graphs of Polynomial Functions	Functions	Graph and describe key features of polynomial functions, and translate between representations.
29	Interpret Functions in Context	Functions	Identify and interpret key features of nonlinear functions in real-world situations.
30	Revealing and Comparing Properties of Functions	Functions	Rewrite quadratic and exponential functions in different forms to reveal key features. Compare and interpret key features of nonlinear functions in real-world situations.
31	Function Transformations	Functions	Recognize the effects of transformations on nonlinear functions. Identify even and odd functions graphically and algebraically.

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32	Inverse Functions	Functions	Find inverses of nonlinear functions for a given domain.
33	Data and Random Sampling	Measurement, Data, & Statistics	Compare theoretical and empirical probabilities of data sets based on models. Select the appropriate method to gather data for real-world situations. Make generalizations based on results from a survey, experiment or observational study.
34	Normal Distribution	Measurement, Data, & Statistics	Use mean and standard deviation to analyze situations and models representing populations fitting a normal distribution curve. Determine subsets of a population based on calculations of area under the normal distribution curve.
35	Probability and Decision Making	Measurement, Data, & Statistics	Recognize the fairness of models. Calculate probabilities of data to draw conclusions about real-world situations.
36	Evaluate and Make Inferences from Data	Measurement, Data, & Statistics	Determine margin of error and make conclusions given proportions related to a population and sample. Draw conclusions from models and data used to report results from real-world experiments, surveys and studies.